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AN EXTENDED PATTERN SEARCH APPROACH TO WIND FARM LAYOUT OPTIMIZATION

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ABSTRACT

An extended pattern search approach is presented for optimizing the placement of wind turbines on a wind farm. The algorithm will develop a two-dimensional layout for a given number of turbines, employing an objective function that minimizes costs while maximizing the total power production of the farm. The farm cost is developed using an established simplified model that is a function of the number of turbines. The power development of the farm is estimated using an established simplified wake model, which accounts for the aerodynamic effects of turbine blades on downstream wind speed, to which the power output is directly proportional. The interaction of the turbulent wakes developed by turbines in close proximity largely determines the power capability of the farm. As pattern search algorithms are deterministic, multiple extensions are presented to aid escaping local optima by infusing stochastic characteristics into the algorithm. This stochasticity improves the algorithm's performance, yielding better results than purely deterministic search methods. Three test cases are presented: a) constant, unidirectional wind, b) constant, multidirectional wind, and c) varying, multidirectional wind. Resulting layouts developed by this extended pattern search algorithm develop more power than previously explored algorithms with the same evaluation models and objective functions. In addition, the algorithm's layouts motivate a heuristic that yields the best layouts found to date.

INTRODUCTION

According to the US Department of Energy, "Increased RD&D efforts and innovation will be required to

continue to expand the wind energy industry" in the United States, where wind power is still a maturing technology [1]. The optimization of wind farm layouts could be considered part of this innovation. The DOE estimates that the demand for energy will increase by 39% in the next twenty years [1] with the intent of supplying 20% of the total electricity demand utilizing wind power. Outdated means of energy development have been attributed to climate change, pollution, and permanent depletion of natural resources. These concerns have led to public demand for cleaner, sustainable energy sources like wind turbine technology. To meet these demands it will be necessary to ensure new wind farm installations are developing as much power as possible, which depends on the wind farm layout's capability to account for local wind conditions and aerodynamic interaction between turbines.

The complexity of the wind farm layout problem lies in the dynamic conditions of the farm site and the modeling available to represent realistic wind patterns and wake interactions. A turbine in wind will develop a turbulent wake that decreases the wind speed downstream. On a wind farm, turbines are typically placed in close enough proximity that the effect of placement in a wake could drastically reduce the effective wind speed to downstream turbines and therefore decrease their power output. This work seeks to develop an optimization algorithm that will consider these complexities and improve upon previously explored methods to provide optimal wind farm layouts.

PREVIOUS APPROACHES

The first attempt at applying computational optimization algorithms to the wind farm layout problem was by Mosetti et

al. [2], utilizing a genetic algorithm. This preliminary study developed the framework that has been continuously used for comparison purposes, including the objective function formula and the use of the Jensen wake model [3]. The Mosetti work used a discretized solution space of 100 x 100 cells and limited the placement of turbines to the center of each cell. The algorithm considers each row of the grid as a binary string, mimicking biological evolution by partnering strings to develop “children” strings and infusing genetic mutation. A similar but improved approach was performed by Grady et al [4] whose algorithm incorporated heuristic knowledge about wind farms and utilized more advanced computational resources. A more recent study (2009) by Wan et al. [5] improved on both previous genetic algorithm approaches by implementing a second optimization phase that allowed the turbines to be moved within their assigned cells. Though these works have advanced the study of wind farm layout optimization, these genetic algorithms may have been hindered by the inherent discretization of their binary chromosomes, which directly affects the final precision of the objects in a layout problem.

Employing a different approach, Marmidis et al. [6] used a Monte Carlo method, which relies on the use of random turbine placement. In this study, turbines were randomly placed on the same 100 x 100 grid used in previous GA approaches and then evaluated, with this process iterating until certain stopping criteria were met. Though Marmidis et al. reported better results than prior approaches, differences in evaluation imply discrepancies in their reported results. Regardless, the described Monte Carlo algorithm exhibits similar precision deficiency as previous algorithms due to its use of a discretized solution space.

It is pertinent to explore applying an extended pattern search (EPS) algorithm to the wind farm layout problem, in that EPS is specifically designed to explore problems with multi-modal solution spaces. The extended pattern search combines the effectiveness of deterministic algorithms with the global exploration capability of stochastic methods, which can lead to increases in efficiency and thus the energy output of the farm. EPS algorithms have been successfully used for 2D and 3D layout of products and packages [7][8]. The goal of this work is to discern the performance of the extended pattern search on a wind farm layout and determine if the results are viable. The following section of this paper will describe the details of the wake modeling (which determines power) and cost modeling. Next, the methodology of the extended pattern search is discussed, followed by a description of the numerical procedure of this particular problem. Lastly, results of this study are presented and compared to previous work and conclusions about the algorithm’s performance are drawn.

WAKE AND COST MODELING

A wake model is a simplified quantitative means of representing the fluid interaction between turbines. The wake developed by a turbine in wind greatly reduces the wind speed immediately behind the rotor, increasing slowly with distance

downstream, as shown in Figure 1. The wake model accounts for wake interactions and develops the effective wind speed – the oncoming wind speed at any turbine after wake speed deficits from upstream turbines have been incorporated.

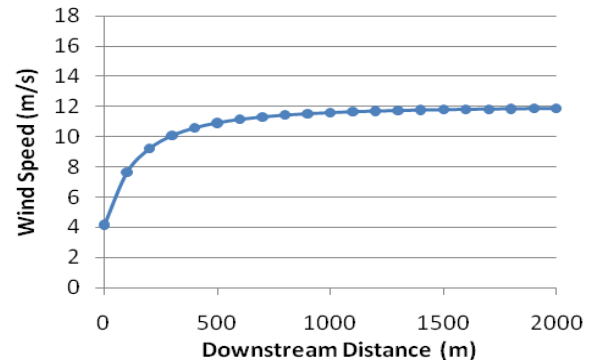


Figure 1: IMMEDIATE SPEED REDUCTION AND EVENTUAL ASYMPTOTIC APPROACH TO INITIAL WINDSPEED (12 m/s) WITHIN A WAKE

This effective wind speed determines the power output of the rotor – generally, the higher the wind speed, the more power a turbine can produce. With the assumption that momentum is conserved inside the wake, a momentum balance is established:

$$\pi r_r^2 v + \pi (r_1^2 - r_r^2) u_0 = \pi r_1^2 u \quad (1)$$

where r_r is the rotor radius, u_0 is the ambient wind speed, v is the wind speed just behind the rotor, r_1 is the effective downstream radius of the wake, and u is the wind speed in the wake downstream of a turbine at distance x , as seen in Figure 2.

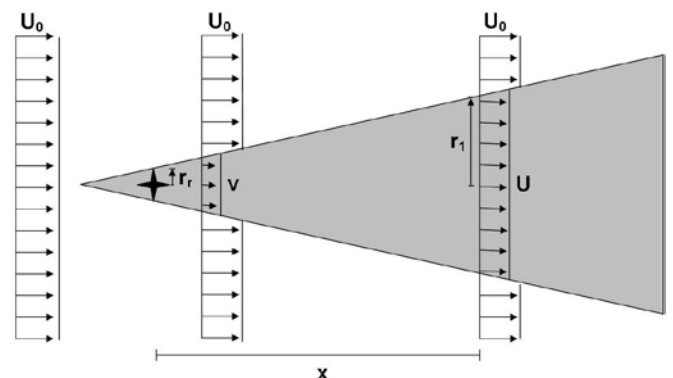


Figure 2: SCHEMATIC OF WAKE MODEL

Assuming a linear relationship between x and r_1 (as indicated by the triangular wake) and employing the theory that assumes the wind speed directly behind the rotor is $\approx 1/3$ of the oncoming wind speed [3], equation (1) can be solved for the downstream wind speed u :

$$u = u_0 \left(1 - \frac{2}{3} \left(\frac{r_r}{r_1} \right)^2 \right) \quad (2)$$

r_1 is related to the turbine rotor radius r_r by a linear relationship with downstream distance x :

$$r_1 = r_r + \alpha x \quad (3)$$

α is known as the entrainment constant and is given by:

$$\alpha = \frac{0.5}{\ln(z/z_0)} \quad (4)$$

where z is the hub height of the turbine and z_0 is the surface roughness of the ground. Formula (3) is used to discern whether a turbine is in the wake of an upstream turbine. Alternatively, a variant but nearly identical formulation for effective wind speed has also been used [2][5]:

$$u = u_0 \left(1 - \frac{2a}{\left(1 + \alpha \left(\frac{x}{r_1} \right) \right)^2} \right) \text{ where } r_1 = r_r \sqrt{\frac{1-a}{1-2a}} \quad (5)$$

where a is the axial induction factor.

For the case of turbines located in multiple wakes, the effective wind speed is calculated by summing the kinetic energy deficits of the individual wakes. The effective wind speed for a turbine in n wakes is calculated using the following expression:

$$u = u_0 \left[1 - \sqrt{\sum_{i=1}^n \left(1 - \frac{u_i}{u_0} \right)^2} \right] \quad (6)$$

These formulae indicate three distinct means of developing the effective wind speed for each turbine: 1) If the turbine has no upstream turbines, the effective wind speed is the ambient wind speed; 2) If the turbine is located in one wake, formulas (2) or (5) are used; and 3) if the turbine has multiple upstream turbines and is therefore located in multiple wakes, formula (6) is used. The effective wind speeds of each turbine are utilized to develop a total power estimate for the N turbines on the farm, given by:

$$P_{tot} = \sum_{i=1}^N 0.3u_i^3 \quad (7)$$

It is also necessary to minimize the cost of turbine installation and use. Mosetti et al. [2] established an expression for estimating the cost of a farm per year, based solely on the number of turbines N . This expression incorporates a cost reduction for the installation of larger- N farms; other factors that may be included are not specifically noted by Mosetti et al.

$$\text{cost} = N \left(\frac{2}{3} + \frac{1}{3} e^{-0.00174N^2} \right) \quad (8)$$

It should be noted that this expression is normalized, where each turbine has a cost of 1.

The objective function for the wind farm layout optimization must maximize the power output of the farm while minimizing cost. Therefore, the objective function is given by:

$$\text{Objective} = \frac{\text{cost}}{P_{tot}} \quad (9)$$

The optimization algorithm will search for the minimum value of this objective function.

EXTENDED PATTERN SEARCH METHODOLOGY

The wind farm layout problem has characteristics that require non-deterministic strategies to effectively solve. The space is multi-modal as was verified by running a deterministic search algorithm with inferior results to our EPS approach. Although various stochastic methods such as genetic algorithms (as used by Mosetti et al. [2] and Grady et al. [5]), simulated annealing [9] or others are feasible, the benefits of the EPS algorithm are described as follows.

Pattern search algorithms are deterministic search methods that use a set pattern of search directions in order to find an optimal placement [10]. Pattern search is a direct search method in that it traverses a sequence of trial solutions, maintaining the best evaluation and comparing it directly to each subsequent evaluation, without the need to evaluate derivatives. At every move in the pattern, the objective function is evaluated and compared to the best prior evaluation, only selecting moves that result in an improved fitness value. Constraints that prohibit the infeasible placement of turbines are enforced with every move. Each move spans a set step size, which is initially defined with respect to the size of the solution space to allow for broad movement. Once every turbine has remained stationary through all pattern directions (in this case, along the coordinate axes) at a given step size, the step size is halved and the search is repeated. The search exits when no possible moves for any turbine have developed an improvement in the objective function evaluation at a minimum allowable step size.

In order to avoid converging on local minima (a common pitfall of deterministic algorithms), extensions are employed that add stochasticity to the system [7].

Understanding of the algorithm is best facilitated by a detailed description of the extended pattern search, including extensions, expressed as flowchart in Figure 3.

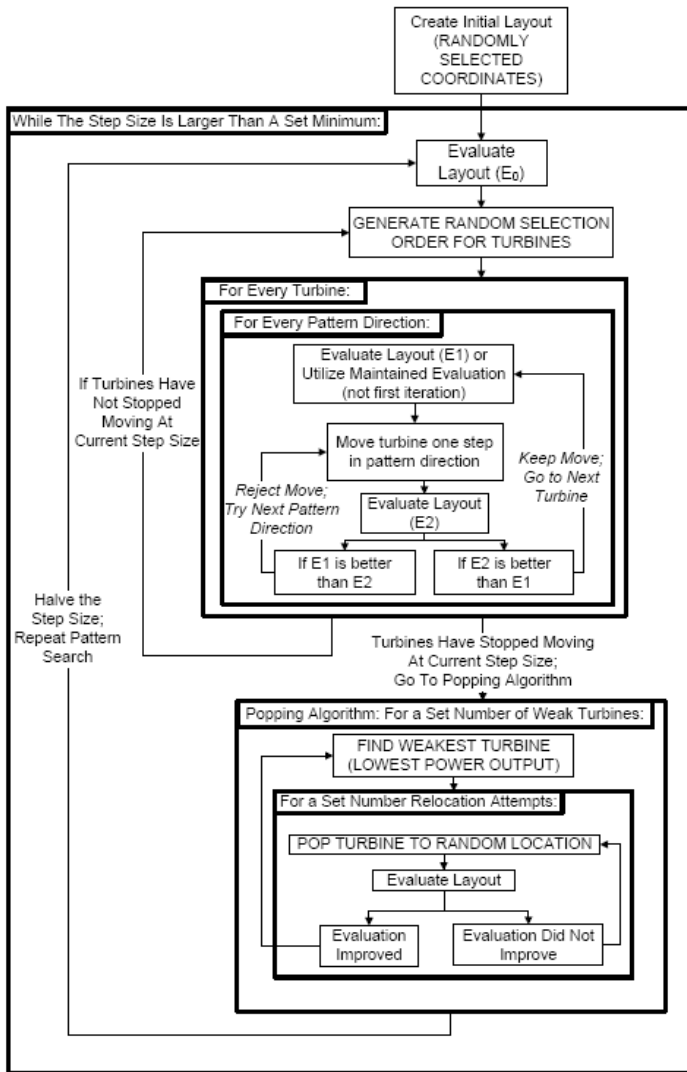


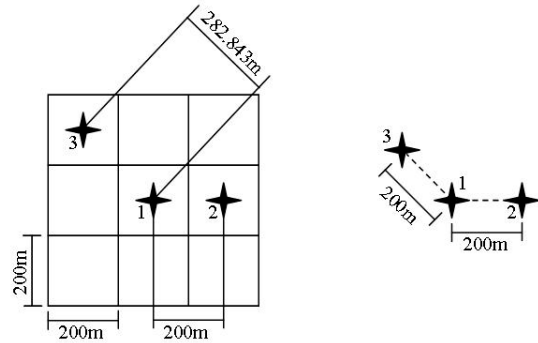
Figure 3: FLOWCHART FOR CURENT EPS

Following the method of Yin and Cagan [7], the extensions to the pattern search are indicated in the flow chart by capital letters. The first extension is the use of a random initial layout. The only restriction on the initial layout is the same constraints applied to every trial move – a turbine cannot be placed within five rotor diameters of another turbine or outside of the solution space. The second extension is the randomized selection order of turbines. This ensures that no turbine’s individual movement is favored over another. The final extension is the popping algorithm that performs once all turbines have stopped moving at the current step size. A number of the weakest performing turbines (based on individual power output) are “popped” to a random location. The new locations are kept if they do not cause interference and improve the evaluation. The popping

algorithm exits after a given number of pop attempts are made or an improved location is found for each of the selected weak turbines.

NUMERICAL PROCEDURE

The extended pattern search algorithm uses a continuous solution space, the benefit of which is made evident by the minimum distance requirement between turbines to avoid interference. In the discretized solution spaces of previous works [2, 4, 6] the five rotor diameter minimum distance is inherently satisfied in the directions along the X and Y axes by locating turbines in the center of each cell. However, turbines in diagonally adjacent cells are actually further apart than the specified minimum distance [Figure 4a]. The continuous solution space allows for turbines to be placed at the true minimum distance in any direction [Figure 4b].



(a) Representation from previous work (b) Current representation

Figure 4: COMPARISON OF SOLUTION SPACES - a) DISCRETIZED, AND b) CONTINUOUS

In this test case, the solution space is 2 km by 2 km, and the turbine geometry consists of a 60 m hub height and a 40 m rotor diameter. Case (a) is that of constant wind speed (12 m/s) and unidirectional wind (from the bottom of the field in the +Y direction). Case (b) also has constant 12 m/s wind speed, but the wind is considered from 36 rotational directions (in ten degree increments) with equivalent probabilities of occurrence. Case (c), the most complex wind case, considers three wind speeds (8, 12, and 17 m/s) and the same 36 rotational directions, and both wind speed and direction are of varying probabilities of occurrence. Considering there are no restrictions on turbine placement (apart from avoiding interference), it is necessary to systematically determine which turbines are located in the wakes of upstream turbines. For each turbine the maximum width of its wake (based on its distance from the back of the field) is found and a rectangular neighborhood of that width is created. The turbines that lie in this wake are then flagged, and the upstream distance between each of them and the wake-producing turbine is calculated. Using equation (3), the actual width of the triangular wake r_1 is determined for each flagged turbine, and only those within this

width at their current downstream distance are further considered to be affected by that wake. The initial step size chosen is 400 m and is halved until reaching a minimum value of 3.125 m. The popping algorithm attempts to relocate the worst performing turbine five times, using up to 10,000 random locations depending on the number of turbines in the layout.

RESULTS AND DISCUSSION

Quantitative comparison between the extended pattern search results and the results of previous approaches were developed, including an efficiency percentage (percent at which the farm's power development represents the maximum amount of power possible). Comparison to previous results are shown, first by indicating the EPS layout of the same number of turbines as those from previous work, followed by the optimal EPS layout for each case. As the EPS has stochastic characteristics, it is not guaranteed to converge on the same solution over multiple runs. Following common practice with such algorithms, the EPS was run ten times, and the best solution is shown here.

Case (a)

Comparisons to the Mosetti et al. Genetic Algorithm approach are compiled in Table 1. Visual representations of the 26-turbine Mosetti et al. layout and an example of a 26-turbine EPS-developed layout included in Figure 5a and Figure 5b, respectively.

Table 1: COMPARISON OF MOSETTI ET AL. AND CURRENT EPS APPROACH- CASE (a)

Result	Mosetti et al.	Mosetti et al. Current Eval	Current EPS
Fitness	0.0016197	0.00158985	0.00148433
Total Power (kW)	12,352	12,583.8	13,478.4
N	26	26	26
Efficiency	91.64%	93.36%	100.00%
Discrepancy		1.870%	

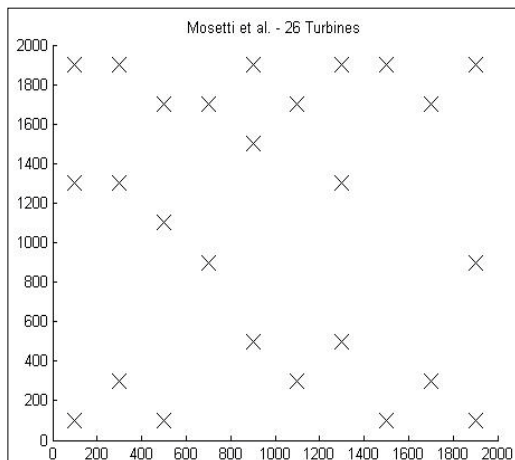


Figure 5a: MOSETTI ET AL. 26-TURBINE LAYOUT – (a)

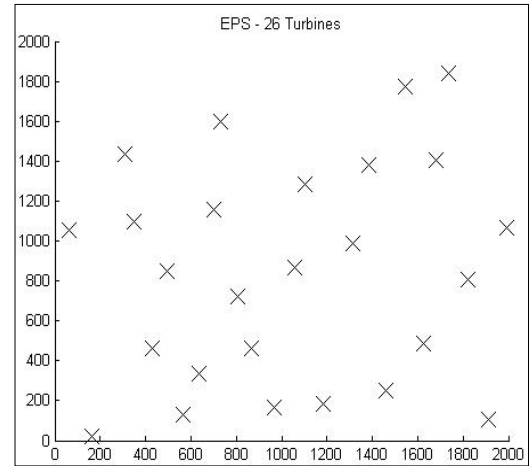


Figure 5b: EPS 26-TURBINE LAYOUT – (a)

Comparison to the Grady et al. Genetic Algorithm approach [4] are compiled in Table 2, with visual representations of the 30-turbine Grady et al. layout and an example of a 30-turbine EPS developed layout included in Figure 6a and Figure 6b, respectively.

Table 2: COMPARISON OF GRADY ET AL. AND CURRENT EPS APPROACH – CASE (a)

Result	Grady et al.	Grady et al. Current Eval	Current EPS
Fitness	0.0015436	0.00152255	0.00142032
Total Power (kW)	14310	14,507.8	15,552
N	30	30	30
Efficiency	92.01%	93.29%	100.00%
Discrepancy		1.377%	

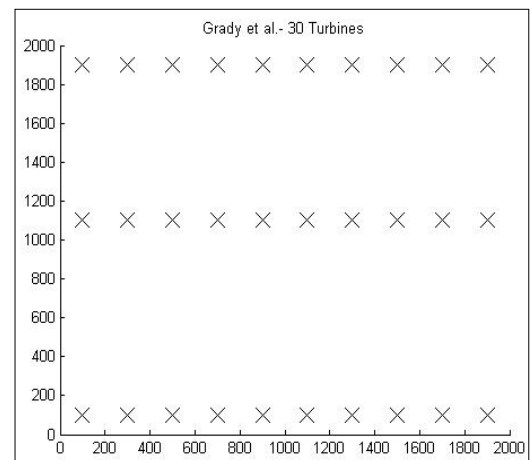


Figure 6a: GRADY ET AL. 30-TURBINE LAYOUT – (a)

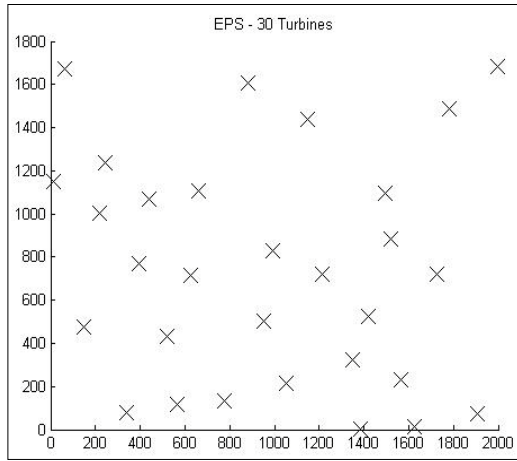


Figure 6b: EPS 30-TURBINE LAYOUT – (a)

The layouts of previous studies were tested using the current wake model evaluation, and as such exhibit a slight discrepancy from the original reported result. For the Mosetti et al. and Grady et al. results, this discrepancy is 1.87% and 1.38%, respectively, indicating convincing consistency between evaluation models. However, the new evaluation of layouts from Marmidis et al. exposes dissimilarity in modeling (a 35.386% discrepancy). Studies of their results indicate a likely error in their reported data (for example, a theoretically impossible efficiency greater than 100% with turbines placed in the wakes of others). Therefore, comparison to this reference will not be included.

For Case (a), the extended pattern search algorithm is able to create multiple 100% efficient layouts for trials that employ up to 43 turbines. Therefore, in comparison to both the 26-turbine Mosetti et al. result and the 30-turbine Grady et al. result, the current extended pattern search algorithm creates more efficient layouts that develop more power.

To determine the optimal number of turbines that should be considered for this farm, a plot of objective function evaluation versus the number of turbines is generated from EPS trials [Figure 7]. The curve marked by circles represent the actual EPS trial data (with a polynomial fit superimposed), and the curve marked by squares indicate theoretical 100% efficient objective function evaluations.

The 4th-order polynomial fit of the objective function evaluation vs. N (number of turbines) curve gives the following equation:

$$y = 7.4205 \cdot 10^{-11} (x^4) - 1.9861 \cdot 10^{-8} (x^3) + 2.0041 \cdot 10^{-6} (x^2) - 8.9108 \cdot 10^{-5} (x) + 0.0027657 \quad (10)$$

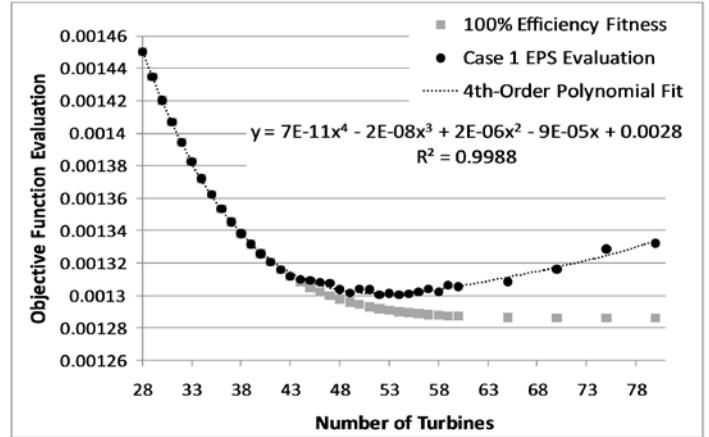


Figure 7: OBJECTIVE FUNCTION EVALUATION VERSUS NUMBER OF TURBINES – CASE (a)

The minimum value of this curve is the estimation of the best objective function evaluation for any number of turbines. The value of x that minimizes this function is $x \approx 52$. Therefore, the estimated optimal result of the extended pattern search is obtained by developing a 52-turbine layout [Figure 8]. This layout develops 26,831 kW of power per year and is 99.534% efficient. Although this layout is less than 100% efficient, the design is still the most cost effective for the power generated based on the overall objective function evaluation.

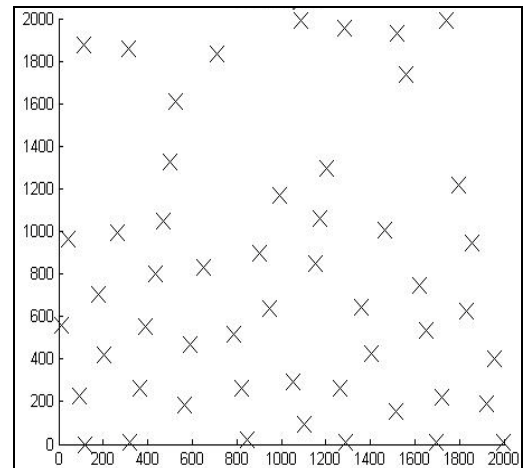


Figure 8: OPTIMAL 52-TURBINE EPS LAYOUT – CASE (a)

For Case (a), The EPS layouts created for larger numbers of turbines (including the 52-turbine layout in Figure 8) reveal distinct patterns of placement; primarily long vertical “strings” of turbines placed on slight diagonals, often forming “W” shapes. Based on the shapes identified in these solutions, a layout was manually developed as a heuristic derived from the EPS algorithm results. A plot of the 100% efficiency objective function value versus the number of turbines [Figure 7] shows an asymptotic approach to a minimum fitness value. It appears that the curve begins this asymptotic behavior near $N=56$,

which indicates that the addition of more turbines above this number do not greatly benefit the objective function evaluation. Therefore the manually developed layout includes 56 Turbines [Figure 9] This layout exhibits 100% efficiency, and develops 29,030.4 kW of power per year.

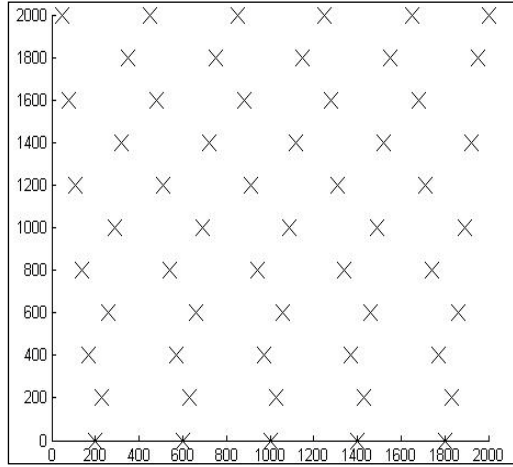


Figure 9: 100% EFFICIENT 56-TURBINE LAYOUT – CASE (a)

Case (b)

The second wind case is also that of constant wind speed (12 m/s), but now includes the more realistic approach of varying the wind onset angle. In 10-degree increments, 36 different rotational angles are considered, which allows for a thorough test of the solution space. The solution for the individual 36 angles are equally weighted and combined. Comparisons of the current EPS algorithm, along with re-evaluation of the Mosetti et al. layout using the current model, are found in Table 3. Visual representations of both the Mosetti et al. Case (b) 19-turbine layout and an example of a 19-turbine EPS layout are shown in Figure 10a and Figure 10b, respectively.

Table 3: COMPARISON OF MOSETTI ET AL. AND CURRENT EPS APPROACH – CASE (b)

Result	Mosetti et al.	Mosetti et al. Re-Evaluation	Current EPS
Objective Function Evaluation	0.0017371	0.00169916	0.00164384
Total Power (kW)	9244.7	9443.5	9761.3
N	19	19	19
Efficiency	93.86%	95.88%	99.10%
Discrepancy		2.10%	

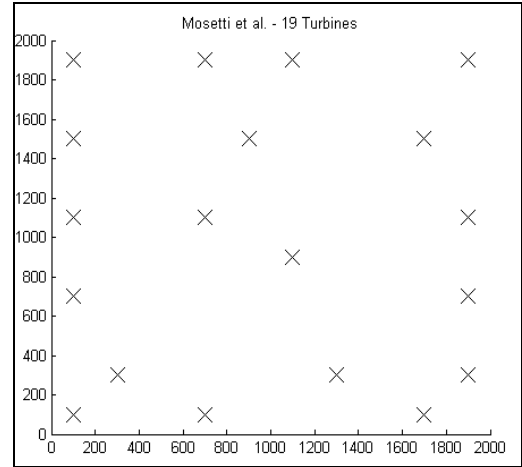


Figure 10a: MOSETTI ET AL. 19-TURBINE LAYOUT – CASE (b)

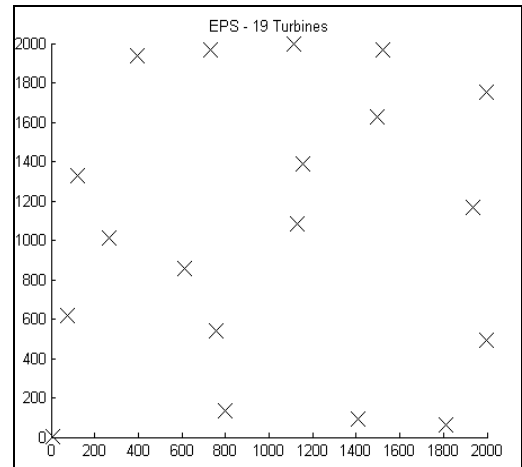


Figure 10b: EPS 19-TURBINE LAYOUT - CASE (b)

The EPS-developed 19-turbine layout for the multidirectional, constant wind speed case shows behavior of turbine migration toward the outer edges of the field. This behavior implies the algorithm is attempting to place turbines as far away from each other as possible to maximize their effective wind speed from all angles.

Similarly, Table 4 shows the Grady et al. result for Case (b), the re-evaluation of the Grady et al. layout through the current model, and an example of an EPS results for the same number of turbines, for comparison purposes.

Table 4: COMPARISON OF GRADY ET AL. AND CURRENT EPS APPROACH – CASE (b)

Result	Grady et al.	Grady et al. Re-Evaluation	Current EPS
Objective Function Evaluation	0.0015666	0.00146355	0.00139159
Total Power (kW)	17220	18394.8	19346
N	39	39	39
Efficiency	85.17%	90.98%	95.69%
Discrepancy		6.40%	

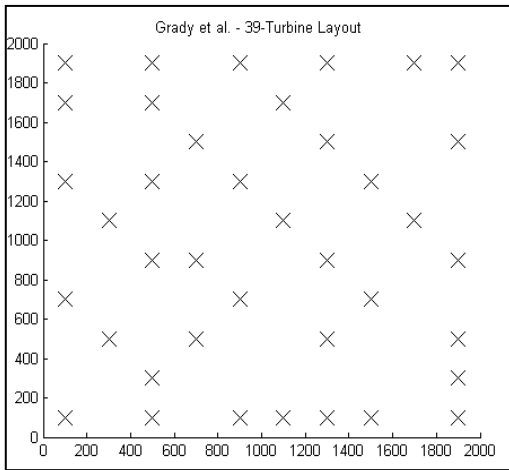


Figure 11a: GRADY ET AL. 39-TURBINE LAYOUT – CASE (b)

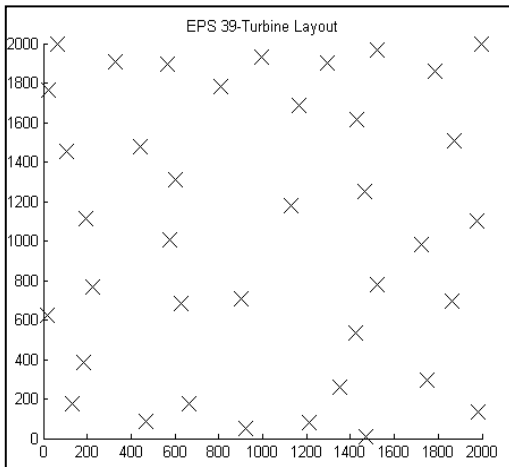


Figure 11b: EPS 39-TURBINE LAYOUT - CASE (b)

From the previous figures (11a and 11b), the behavior of the turbines for this higher- N layout differs from that of the previous 19-turbine layout. Instead of primary migration toward the outside of the field, the turbines are generally avoiding being placed in straight lines from all directions. The optimal EPS layout can be found using the same means as presented in the first case. Figure 12 shows the 4th-Order polynomial fit to the data obtained from performing EPS Case (b) trials.

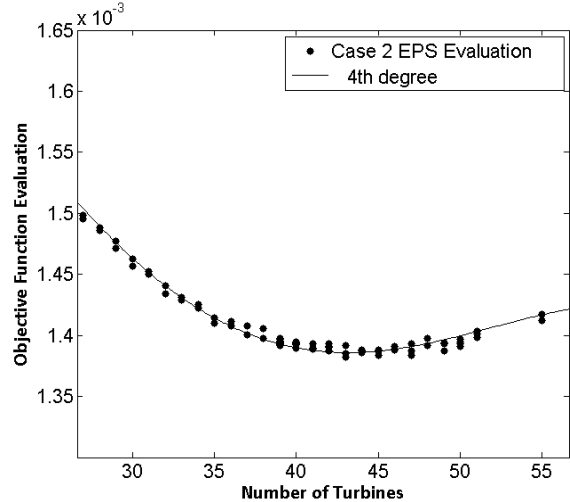


Figure 12: OBJECTIVE FUNCTION EVALUATION VS NUMBER OF TURBINES - CASE (b)

From the 4th-Order polynomial fit to the Case (b) trial data, the minimum of the curve was found to indicate a likely number of turbines that would result in the minimum objective function evaluation. This value is found to be approximately 43 turbines. An example of a 43-Turbine EPS layout for Case (b) is shown in Figure 13.

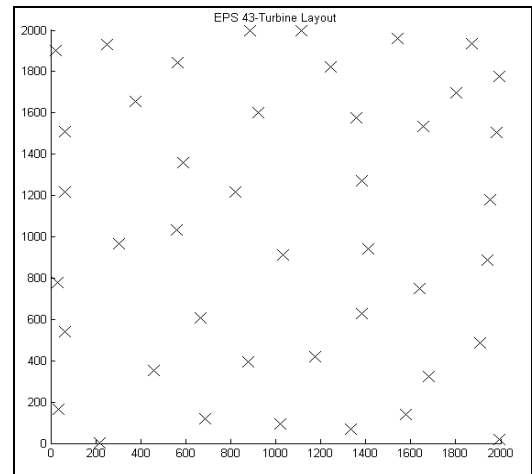


Figure 13: OPTIMAL 43-TURBINE EPS LAYOUT - CASE (b)

This 43-Turbine Case (b) EPS layout is 95.09% efficient and develops 21195.7 kW.

Case (c)

The final case explored is multidirectional, varying wind speed. The three possible wind speeds are 8 m/s, 12 m/s, and 17 m/s. As with Case (b), all 36 angular directions are considered (again occurring in 10-degree increments), however, in Case (c) these directions and the wind speeds for each direction are weighted. The sum of each wind direction and

wind speed case A bar graph depicting the weighting for these is given in Figure 14.

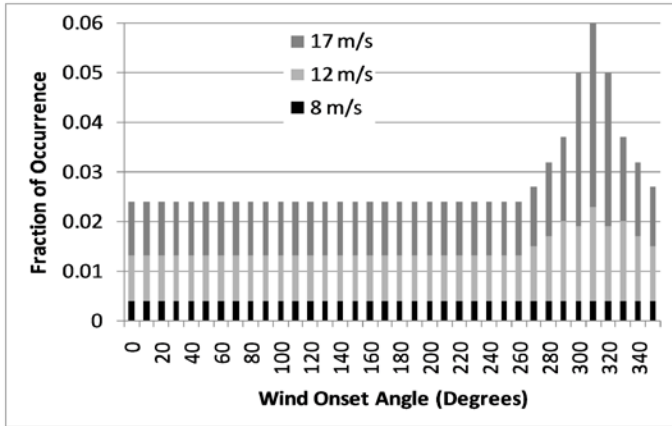


Figure 14: BAR GRAPH OF WEIGHTING FRACTIONS - CASE (c)

The multidirectional, varying wind speed case presents the opportunity to gauge the performance of these algorithms in a more realistic wind environment. Table 5 summarizes this performance for the 15-Turbine Mosetti et al. result for Case (c), as well as an example of a 15-Turbine EPS result. Plots of both of these layouts follow Table 5 (Figures 15a and 15b)

Table 5: COMPARISON OF MOSETTI ET AL. AND CURRENT EPS APPROACH- CASE (c)

Result	Mosetti et al.	Mosetti et al. Re-Evaluation	Current EPS
Objective Function Evaluation	0.00099405	0.000984478	0.000966375
Total Power (kW)	13460	13591.2	13845.8
N	15	15	15
Efficiency	96.66%	97.60%	99.43%
Discrepancy		0.97%	

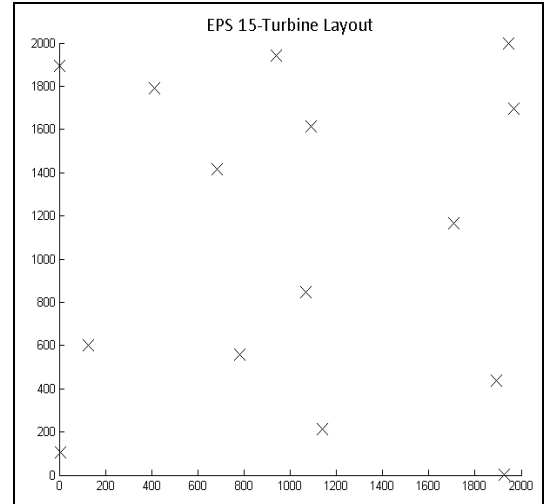


Figure 15b: EPS 15-TURBINE LAYOUT - CASE (c)

As it is difficult to see any patterns emerge from trials with smaller numbers of turbines, a stronger understanding of the effects of multidirectional varying wind can be seen in comparing the results of the Grady et al. and the current study. These results are summarized in Table 6, and figures representing the two layouts follow (Figures 16a and 16b).

Table 6: COMPARISON OF GRADY ET AL. AND CURRENT EPS APPROACH - CASE (c)

Result	Grady et al.	Grady et al. Re-Evaluation	Current EPS
Objective Function Evaluation	0.00080314	0.000828565	0.000778265
Total Power (kW)	32038	32491.9	34591.9
N	39	39	39
Efficiency	88.49%	89.74%	95.55%
Discrepancy		1.39%	

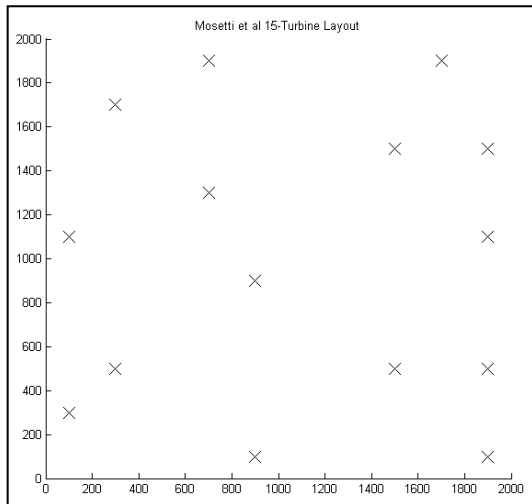


Figure 15a: MOSETTI ET AL. 15-TURBINE LAYOUT - CASE (c)

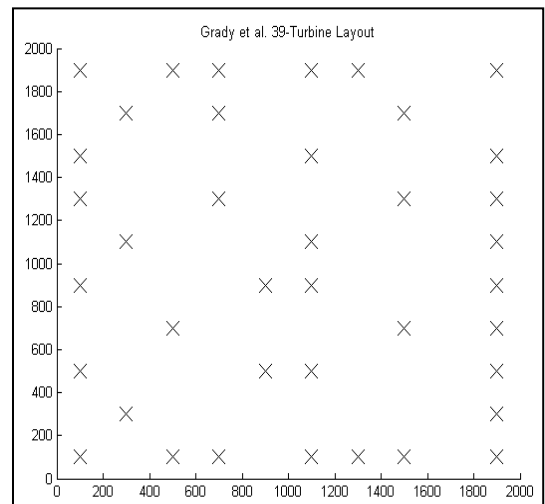


Figure 16a: GRADY ET AL. 39-TURBINE LAYOUT - CASE (c)

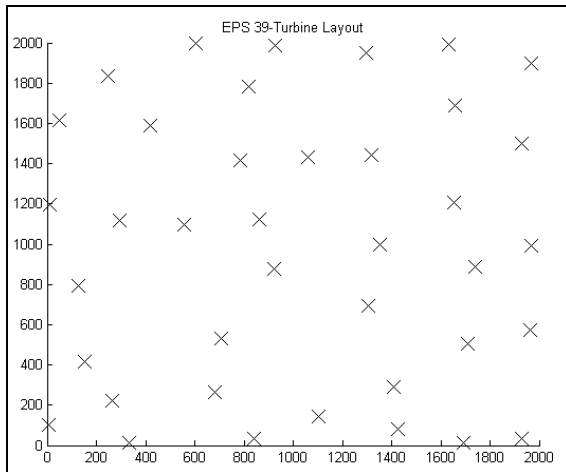


Figure 16b: EPS 39-TURBINE LAYOUT - CASE (c)

As in Case (b), the optimal number of turbines suggested for the EPS layout was 43. An example of a 43-Turbine EPS layout for Case (c) is included below as Figure 17. This layout is 94.19% efficient and develops 37600.9 kW.

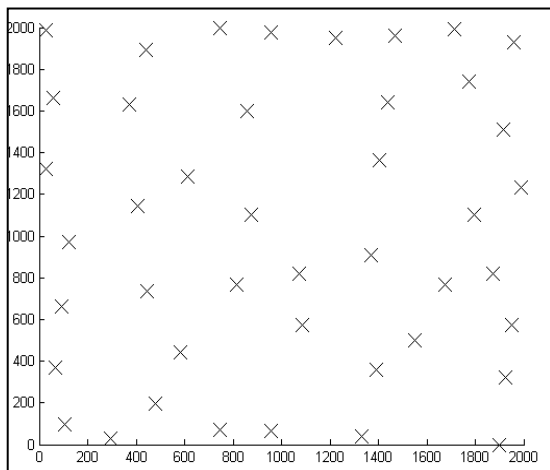


Figure 17: OPTIMAL EPS 43-TURBINE LAYOUT - CASE (c)

CONCLUDING DISCUSSION

From these results, it is clear that the extended pattern search algorithm described is promising for these wind cases. It exhibits higher efficiency and develops more power than layouts presented in previous literature. The extended pattern search is also successful in that the conclusions drawn from resulting layouts influenced the manual development of high- N 100% efficient layouts. For Cases (b) and (c), the EPS was able to generate slightly higher efficiency and higher power capable layouts, however, the behavior indicated by the turbine movement in these layouts suggests further exploration in heuristic advantages for more complex wind cases. In order for the extended pattern search to be a viable choice for wind farm layout optimization, further studies must be conducted that utilize realistic wind data. Because the layout problem is multi-

modal and a stochastic algorithm is employed, the EPS generated better performing layouts than the literature, though likely not the global optima. This allows for future development of heuristics and other extension to this work.

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